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A stress inversion procedure for polyphase fault/slip data sets

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Abstract—Palaeostress estimation from striated fault data is frequently frustrated by the fact that natural fault data are heterogeneous in the sense that they cannot satisfactorily be explained by reactivation brought about by a single stress tensor. In many instances there is clear evidence to show that striation data can record the effects of multiple stress events. Any attempt to find the tensor which best fits (or explains) such dynamically mixed data sets risks determining a spurious stress state which is some form of compromise between the real stress states corresponding to different tectonic phases. To avoid this problem a strategy is proposed here which involves an initial separation of the raw data into coherent sub-sets prior to formal stress inversion. This separation is performed by assigning attributes to each fault which describe the fault's compatibility with trial stress tensors. Using these attributes faults can be grouped into dynamically-homogeneous families using the statistical techniques of cluster analysis.

INTRODUCTION

Since the pioneering paper by Carey & Brunier (1974) several computational methods have been devised for estimating palaeostresses in brittle tectonic environments from field measurements of fault planes and associated slip lineations (e.g. Angelier 1979, Etchecopar *et al.* 1981, Michael 1984, Hardcastle & Hills 1991). The majority of these are founded on three critical assumptions:

1. Fault slip recorded by the measured striae was parallel to direction of resolved shear stress on the fault.
2. Slip on the different faults is mutually independent.
3. The stresses which induced slip on the faults can be considered homogeneous on a macroscopic scale.

Although many of the above techniques have become widely accepted and are often applied for palaeostress analysis, the highly restrictive nature of the assumptions means that (a) the methodology is only appropriate in certain geological situations, and (b) the results produced should always be critically evaluated.

The third assumption listed is particularly limiting since field evidence suggests that instances of multiple phases of fault reactivation are commonplace in all but the youngest of sediments. If data collected from areas which have been subjected to multiple stress events are analysed using standard techniques for stress analysis, the obtained results are likely to be of doubtful value, giving at best a calculated stress configuration which is some sort of compromise between the different real stress tensors represented in the data. Under such circumstances the exercise degenerates from one of stress estimation to merely a vague statistical description of the fault data. This paper addresses the vital issue of analysing stresses from mixed data sets and proposes a new approach involving the separation of the sample of faults into homogeneous sub-sets prior to stress inversion.

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EXISTING APPROACHES TO ANALYSIS OF MIXED DATA SETS

In several routines currently available for stress inversion of fault/slip data (Carey & Brunier 1974, Angelier 1979, Etchecopar *et al.* 1981, Armijo *et al.* 1982) the stress tensor is sought which best 'explains' the observed movement vectors on the measured faults (see Angelier 1994 for a comprehensive review of different methods for seeking the tensor). The success with which a given tensor accounts for the recorded slip direction indicators is normally quantified as the sum of the squares of some measure of deviation between the observed striations and those predicted from the stress tensor being considered. Once a best tensor is found the most deviant faults can be identified and, if necessary excluded from further analysis.

Although this approach may be acceptable for data consisting for faults whose slip was induced by a common stress state, it is an unsound strategy for dealing with dynamically-mixed data sets. For such a method to succeed it needs to correctly identify outliers; those fault data perhaps originating from secondary stress phases. As Will & Powell (1991) point out, the presence of such outliers can have a profound effect on the ultimate best-fit tensor which is calculated. If this estimated tensor does not correspond to a real one then the identification of outliers will usually also be incorrect. Will & Powell's (1991) suggested remedy, to select the tensor which minimises the median of the squares of the striation misfits, may help to alleviate the problem in cases where a single set of faults is contaminated by a relatively minor proportion of rogue faults. It cannot however solve the problem of a truly mixed data set involving representatives from two (or more) tensors since the *best-fit stress tensor* concept is meaningless in that context. The same limitation is possessed by similar methods using different criteria for quantifying best-fit (Michael 1984, Gephart & Forsyth 1984, Gephart 1990).

A similar drawback is associated with the procedures used by Galindo-Zaldivar & Gonzalez-Lodeiro (1988) and Hardcastle (1989) which use a grid-search method of stress inversion for the analysis of heterogeneous fault data samples. This involves considering in turn a large number of different tensors in order to discover those which are capable of explaining a high proportion of the orientation data taken from faults in the field. The tensors which explain the greatest number of faults, i.e. produce deviations lower than a prescribed threshold, are assumed to be favoured candidates for the real palaeostresses. However this assumption has never been validated. Again it is probable that the tensors located by this process are hybrid ones with properties which position them between the real geological solutions.

The above discussion suggests that a strategy which allows faults to be assigned into sub-sets prior to stress inversion is likely to be more satisfactory. Simón-Gómez's (1986) graphical method (see also the refinement of it by Fry 1992) is able to identify groups of related faults. Simón-Gómez's Y–R diagram is a plot of orientation and stress ratio of all stress tensors compatible with a given striated fault. On the Y–R diagram each fault is shown as a curve, and faults belonging to the same sub-set are identified by a common intersection of their respective curves. However, because of the 4-dimensional nature of the problem, only special stress configuration (with one principal stress axis vertical) can be dealt with by this method.

Kleinspehn *et al.* (1989) suggest a way of extracting multiple stress tensors from a fault population which involves first identifying the latest stress tensor from an analysis of faults present in stratigraphically higher units. This approach can be adopted only when the period of sedimentation spans several tectonic events.

Angelier & Manoussis (1980) and Huang (1988) describe procedures for the automatic separation of fault sub-sets. These involve assumptions which make them valid only for the treatment of Andersonian faults, i.e. faults with special, symmetrical orientation with respect to the principal stress axes.

Like the last-mentioned routines, the method we propose in this paper for the separation of sub-groups from heterogeneous collections of data utilises the statistical procedures of cluster analysis. In philosophy however our method is quite different and is not limited to the treatment of conjugate faults but can be used on data from reactivated faults with general orientations.

THE NEW STRATEGY

The method of fault set separation adopted here is based on cluster analysis, a statistical technique for identifying groups of entities from within a sample. The analysis is performed in four stages.

(a) Each fault plane together with its associated slip direction in the sample is described in terms of a number of variables relating to its orientational properties and its compatibility with a large number of potential stress tensors.

(b) All possible pairs of faults are compared on the basis of the above variables and their degree of similarity expressed quantitatively.

(c) The close similarity between certain pairs of faults is used as a basis of the nucleation of fault sub-sets or clusters which grow progressively by the merger with other clusters. These clusters finally constitute the dominant sub-sets consisting of faults with a high degree of mutual dynamic compatibility.

(d) These sub-sets are analysed separately using one of the existing stress inversion routines.

Defining fault attributes

The input data from each fault are considered in relation to a large number of different stress tensors. For each tensor, a calculation is made of the ideal orientation of the striation on a plane with the orientation of a measured fault plane. Mutual compatibility between the tensor and the fault is determined on the basis of angular difference between the calculated striation and the actual (measured) one. The tensor is considered to 'fit' the fault if the deviation in striation pitch does not exceed a prescribed angle of tolerance (α). In this way each fault can be described by a string of binary attributes referring to its compatibility (fit or non-fit) with a variety of tensors (Table 1).

We appreciate that this criterion for matching faults to stress tensors is simple; replacing it with a more sophisticated measure of misfit (e.g. one that takes into account the relative magnitudes of shear stress on the fault into account) will not affect the general philosophy of the method described here.

The tensors used for the purpose of defining fault attributes need to meet certain requirements if they are to produce a set of characteristics which are not biased towards certain faults. The set of tensors used have to (a) be variable in terms of both axial orientations and shape factor, (b) possess isotropy as a set with respect to these properties, and (c) be large in number.

To obtain a set of variable tensor orientations, σ_1 orientations were chosen according to a spherical grid pattern with plunges incrementing in angular intervals of β. To ensure isotropy of these directions the azimuth of the plunge was varied as a function of the plunge angle. For each selected σ_1 orientation a variety of σ_3 directions is produced by incremental rotations through an angle β about the σ_1 axis. The angle β is a measure of the grid spacing and controls the total number of tensors exam-

Table 1. (a) Assigning descriptive attributes to each fault on the basis of its compatibility with a large number of tensors. Each fault is given binary scores which are later used for comparing fault pairs; + = fault fits the tensor, − = fault does not fit tensor. (b) The comparison of faults using their dynamic attributes: a_{ij} is the number of cases where faults i and j both fit a tensor, d_{ij} is the number of cases where neither fault fits a tensor. S_{ij}, the coefficient of similarity defined in equation (1) for a tolerance angle, α, of 30° and a number of tensors $n = 7$, expresses the degree of similarity between a pair of faults, fault i and fault j.

(a)

	Tensor 1	Tensor 2	Tensor 3	Tensor 4	Tensor 5	Tensor 6	Tensor 7
Fault 1	+	−	+	+	−	−	−
Fault 2	+	+	+	+	−	−	+
Fault 3	−	−	+	−	+	+	+
Fault 4	−	−	−	−	+	+	+

(b)

Fault pair 1 & 2: $a_{12} = 3$, $d_{12} = 2$, $N = 7$, $\alpha = 30°$, $S_{12} = 0.61$
Fault pair 1 & 3: $a_{13} = 1$, $d_{13} = 1$, $N = 7$, $\alpha = 30°$, $S_{13} = 0.17$
Fault pair 1 & 4: $a_{14} = 0$, $d_{14} = 1$, $N = 7$, $\alpha = 30°$, $S_{14} = 0.0066$
Fault pair 2 & 3, etc.

ined. For each axial orientation, 20 tensors are examined, each with a different shape factor $\phi = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ (see Angelier 1994) covering the range from $\phi = 0$ (axial compression) to $\phi = 1$ (axial extension).

With regard to the choice of grid spacing β, a value is recommended which is as low as possible, though computer time may place a practical lower limit on this. To increase the computing efficiency the properties of the *inverse stress tensor* are utilised in the calculations (see Appendix). For any given tensor (σ_1, σ_2, σ_3) the corresponding inverse is defined as one which is coaxial with it but with σ_1 and σ_3 interchanged and with a ϕ value (ϕ') equalling $(\sigma_1 - \sigma_2)/(\sigma_1 - \sigma_3)$. That is ϕ', $= 1 - \phi$. It can be readily shown (see Appendix) that any given tensor and its inverse produce shear stress directions on a given plane which are parallel but opposite in sense. This means that once the calculation of the ideal striation orientation on a fault has been performed for a given tensor, the calculation for a second tensor, its inverse, is trivial. This tactic effectively doubles the numbers of tensors that can be examined in a given period of computer time and reduces the effective grid spacing from β to $\beta/\sqrt{2}$. Using this device and an IBM-compatible machine with 486-33 processor, a sample of 30 faults takes some 90 minutes to analyse for an effective grid spacing of 10 degrees (40,000 tensors examined).

The computer program FLTCLAN, details of which are to be published separately, computes ideal slip directions for each fault and each tensor using the algorithm of Hardcastle & Hills (1991). At the end of this stage several thousand binary variates have been calculated for each fault. These allow the comparison of the faults in a manner described below.

Defining similarity between the sampled faults

The next stage is to express the degree of similarity between all possible fault pairs in the sample from a comparison of their attributes. Everitt (1980) suggests various indices of similarity appropriate for binary variables. We have devised a coefficient of similarity which is more appropriate to this problem. This is quantified by a coefficient of similarity S_{ij}, calculated from

$$S_{ij} = [a_{ij} + wd_{ij}]/[n - (1 - w)d_{ij}] \quad (1)$$

where:

a_{ij} = number of tensors that fit faults i and fault j,
d_{ij} = number of tensors which fit neither fault i nor fault j,
α = threshold of angular deviation (in radians) between observed striation pitch and that predicted for a trial tensor used to decide on compatibility,
w = $1/[(\pi/\alpha) - 1]^2$, and
n = total number of tensors examined.

The quantities a_{ij} and d_{ij} are both expressions of closeness, but d_{ij} is a weaker descriptor because, depending on the acceptable angle of deviation (α), failing to fit is a fault attribute which is easier to acquire than fitting. The weighting terms involving w in equation (1) above allow for this. S_{ij} values range from 0, for pairs of faults that are complete opposites, to 1 for complete similarity.

The mutual similarity of faults becomes apparent when a similarity matrix is constructed from S_{ij} coefficients. In the illustrative example of Fig. 1(a) it is clear that fault 5 is more closely allied to fault 3 and fault 4 than to fault 1 and fault 2. To effect a separation of faults into natural sub-groups requires the systematic approach described below.

Organising faults into sub-groups

There are several possible alternative strategies for using the data on pair-wise similarities to decide on the division of the sample into sub-groups or clusters (see Everitt 1980, pp. 23–58). The one adopted here produces excellent results in trials using artificial and natural heterogeneous data sets produced by mixing of fault data sets compatible with known tensors.

To effect the division of N faults into sub-sets a

(a) **Similarity matrix, S_{ij}.**

Fault					
	1	2	3	4	5
1	1.00	0.55	0.08	0.08	0.07
2	0.55	1.00	0.08	0.08	0.07
3	0.11	0.08	1.00	0.31	0.53
4	0.07	0.08	0.51	1.00	0.28
5	0.11	0.07	0.53	0.28	1.00

(b) **Ranked coefficients of fault pairs.**

Ranked	Coefficients	Fault pair
1	0.55	1,2
2	0.53	3,5
3	0.31	3,4
4	0.28	4,5
5	0.11	1,3
etc		

(c)

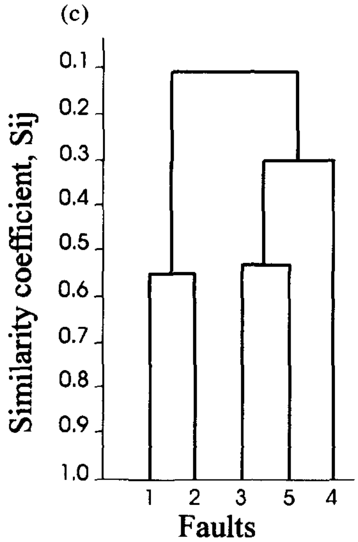


Fig. 1. The construction of a dendrogram in a simple example of 5 faults. (a) Matrix of similarity coefficients, S_{ij}. (b) The ranked list of S_{ij} values to express closest similarities between pairs. (c) Dendrogram constructed from the ranked coefficient. The order in which clusters merge is controlled by the ranked similarity coefficients, i.e. the cluster including faults 1 and 2 unite first because their similarity S_{12} is the highest (0.55). The faults 1, 2 belong to one group; 3, 4, 5 to the other.

dendrogram is constructed (Fig. 1c). This is done by starting with *N* clusters, each consisting of a single fault. Clusters grow by incorporating others. The sequence of amalgamation of clusters is governed by the S_{ij} coefficients from the similarity matrix arranged in ranked order (Fig. 1b). The most similar pair of faults, corresponding to the highest coefficient, are brought together into the same cluster by the merger of the clusters to which each belongs. The single linkage dendrogram, an inverted tree diagram, is a graphical record of the sequence of mergers. An average linkage dendrogram merges clusters on the basis of the average similarity between members of different clusters. This latter variant is preferred here because it prevents the coalescence of disparate groups by the fortuitous similarity of a single pair of faults.

Although ultimately the process leads to the formation of a single group, the branching pattern of the dendrogram serves to highlight important associations of faults and their hierarchy. The first-order branches of the inverted tree indicate the membership of the dominant fault sub-sets.

Stress inversion from sub-sets

Once the heterogeneous sample of faults has been separated into homogeneous sub-sets by the application of the above procedure, palaeostresses can be determined from each of the sub-samples using one of the several existing routines for fault-slip analysis which are well-suited to the treatment of homogeneous data-sets (e.g. Etchecopar *et al.* 1981, Lisle 1988, Angelier 1990).

TESTING OF THE METHOD

In order to test the validity of the method we have examined its performance when analysing natural data

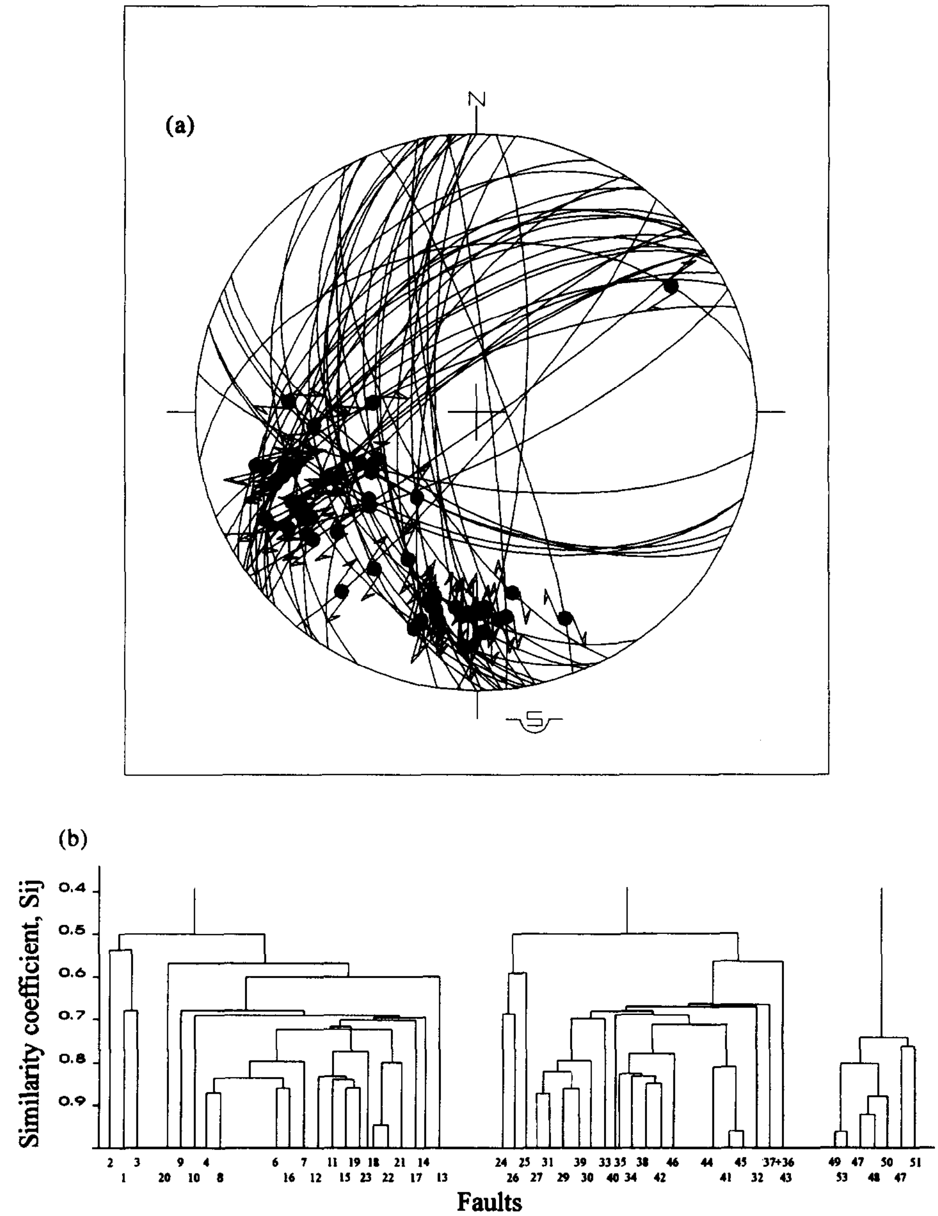


Fig. 2. Automatic fault separation (*Example* 1). (a) Heterogeneous data-set composed on 3 components (23 faults from the Cirocha strike-slip zone, Carpathians, 23 faults formed by rotation of latter by 60°, 7 faults from Bridgend, S. Wales). (b) The fault groupings distinguished by cluster analysis. The main 3 sub-groups correspond exactly to the subsets used to construct the data set.

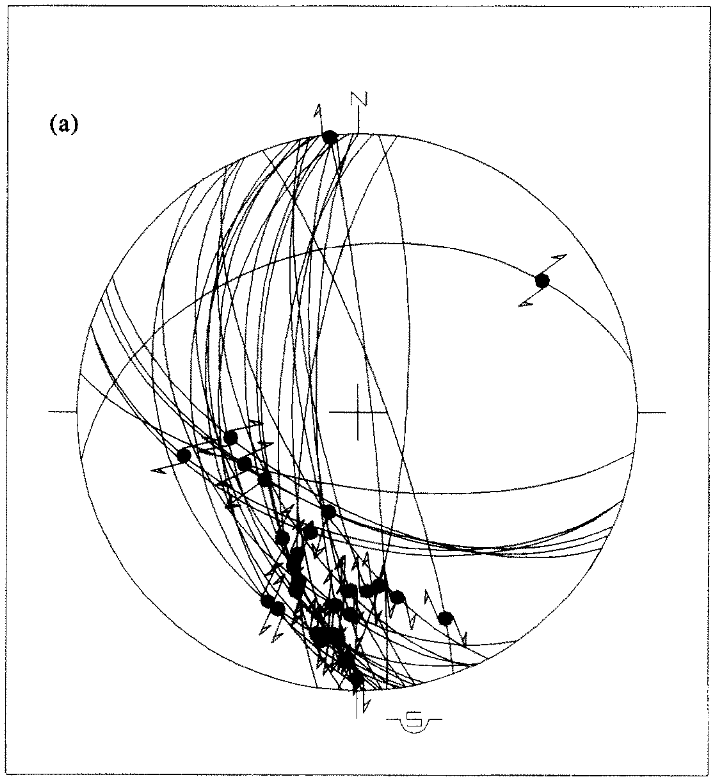
sets which have been artificially mixed in order to simulate the effects of multiple stress events.

Example 1. This data-set (Fig. 2a) consists of a sub-set of 23 faults measured in the Cirocha strike-slip zone in the Carpathians, a second subset (23 faults) produced by the rotation of the first sub-set about a vertical axis through an angle of 60° and a third subset of 7 faults from an extensional (rifting) environment recorded at Trwyn-y-Witch, Bridgend, South Wales. Before being combined these three subsets were separately analysed by grid-search stress inversion and the fault striations adjusted so as to accord with the best-fit tensor of each sub-set.

The combined data-set was used as input for the program FLTCLAN using a grid spacing $\beta = 20°$ and threshold angle $\alpha = 30°$. The results of the cluster analysis are shown as a dendrogram in Fig. 2(b). The three primary branches of the tree indicate the correct sub-set membership exactly.

If this had been a real data-set however we would clearly not have known the number of tensors involved. Our strategy in such circumstances would be to over-divide the sample into more subsets and, by computing tensors for each, determine whether these groupings are meaningful. Sub-groups could be merged if the tensors they yield are not significantly different.

Example 2. This sample consists of two component groups; the 23 faults from Cirocha and 7 faults from



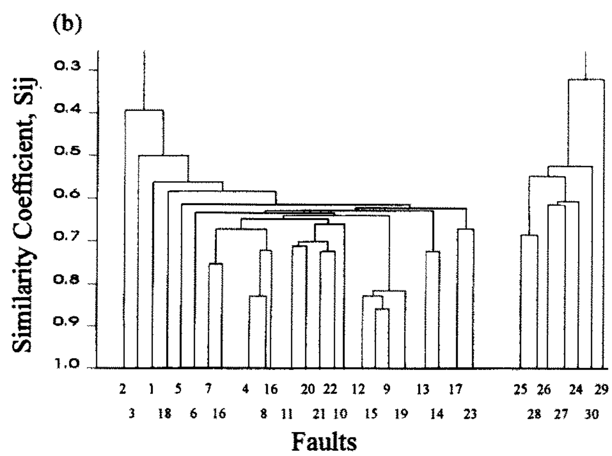


Fig. 3. Automatic fault separation (*Example* 2). (a) Heterogeneous data-set constructed by combining data from Cirocha zone (23 faults) and Trwyn-y-witch sites. (b) Results of the separation procedure revealed by a dendrogram.

Trwyn-y-Witch used in Example 1 (Fig. 3a). Values of $\alpha = 30°$ and $\beta = 20°$ were again used. These data were not adjusted in the manner of the first test data-set and so still included several natural sources of error. For this reason the average degree of similarity between pairs is lower than in Example 1; this results in mergers occurring higher in the dendrogram. In spite of this, the separation routine was able to correctly identify the two sub-samples (Fig. 3b).

Example 3. This consists of four samples of striated faults collected from the Cumberland Plateau thrust zone, southern Appalachians by Steven Wojtal and Jonathan Pershing. The geographical and geological setting of the samples (CPLOW1, CPLOW2, CPMID1 and CPMID2) together with a palaeostress analysis of them is described in Wojtal & Pershing (1991). Their analysis was based on the technique of Etchecopar *et al.* (1981), a computer procedure which instead of estimating the best-fit tensor for the whole sample, allows the user to specify the percentage of faults to be used. However a drawback of this approach arises when heterogeneous data sets are handled, because the user is unlikely to know the fractions of the samples attributable to the different tensors represented.

Cluster analysis was carried out on the samples. Sample CPLOW2 produced a typical dendrogram (Fig. 4a). The maximum number of groups that can be distinguished is limited by the fact that subgroups of faults suitable for stress inversion have a minimum of size of around 10. Seven groups were distinguished initially, though of these only three (labelled a, b and c) possessed sufficient faults for stress estimation. Groups a and c yield well-defined tensors (Fig. 4b), with theoretical and observed striations misaligned by an average 9 and 10° respectively. For group b the corresponding tensor is

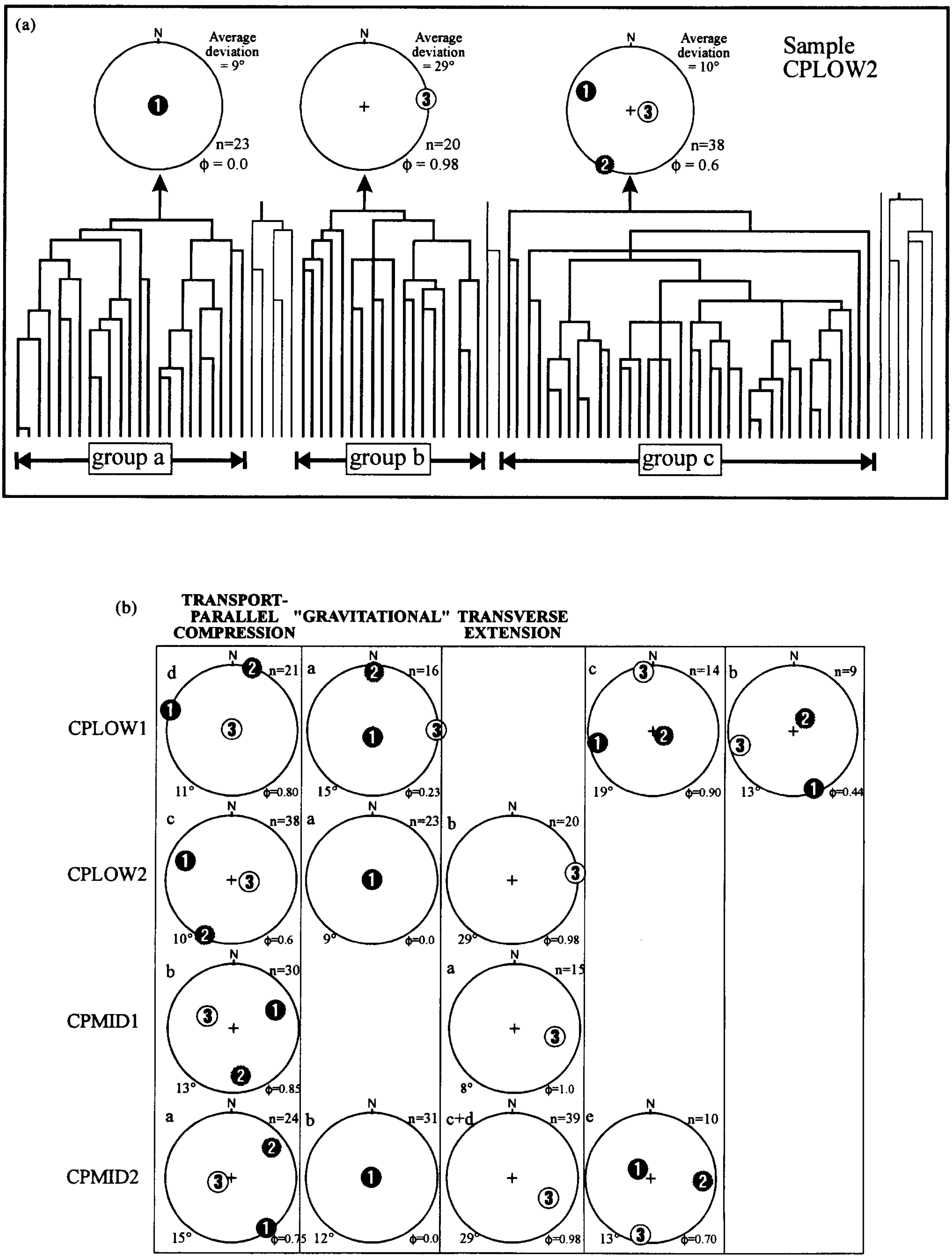


Fig. 4. Automatic fault separation (*Example* 3). (a) Analysis of striated faults (sample CPLOW2), Cumberland Plateau thrust zone, near Dunlap, Tennessee (data supplied by Prof. Steven Wojtal). The dendrogram reveals 3 groups of importance (a, b and c). The corresponding tensors indicate (c) transport-parallel compression, (a) vertical uniaxial compression and (b) transverse uniaxial extension. (b) Summary of the cluster analysis of 4 samples of striated faults (CPLOW1, CPLOW2, CPMID1, CPMID2) collected by Wojtal & Pershing (1991). Three tensors are distinguished in three out of four samples. The transverse extension tensor is the least well defined with high mean angular deviations, see lower left of each stereogram. n = Sub-sample size; ϕ is the stress ratio.

poor, with an average deviation of 29°. This latter group also has a different signature on the dendrogram, characterised by linkage of clusters only at a late stage.

When all four samples are analysed and the obtained tensors compared (Fig. 4b), three tensors are found to be common to three out of four samples. One of these equating to transport-parallel compression corresponds to the first tensor identified by Wojtal & Pershing (1991). Another tensor differential here can be referred to as gravitational (a vertical uniaxial compression with low ϕ) and matches fairly well the second tensor of Wojtal & Pershing (1991). The third is an orogenetically-transverse extension (uniaxial extension, high ϕ) tensor of lesser quality and not distinguished by Wojtal & Pershing.

The degree of success with which the method is able to sort out heterogeneous fault sets will in general depend on (a) the 'difference' between the tensors represented in the sample in terms of axial orientation and ϕ value, and (b) the quality of the fault/striation data in their degree of fit to a component tensor. Fortunately the success of the inversion process does not depend on a perfect partition of the fault clusters; existing routines for stress inversion are designed to cope with a small proportion of rogue data.

CONCLUSIONS

Striation analysis has proved itself in recent years to be a powerful tool for structural studies in brittle tectonic regimes but the inability of present methodologies to handle, in a satisfactory manner, fault data originating from multiple stress events places severe limits on the situations in which the analysis can be sensibly applied.

Traditional techniques used to determine individual stress tensors represented in mixed fault/slip datasets are likely to suffer from problems akin to those encountered when trying to recognize the parameters of Gaussian distributions from mixed samples taken from two or more such distributions. Strategies can easily lead to spurious, hybrid solutions which in turn hinder the correct separation of the data into coherent classes.

The new procedure suggested here for tackling the problem involves describing each fault in the sample by a set of variables which allow sub-sets of faults to be distinguished. The variables chosen to describe the faults are not the faults' simple orientational parameters (e.g. strike, dip, pitch of striation) but are *dynamic* attributes relating more directly to the stress tensors they are compatible with.

Hierarchical cluster analysis in which groups are identified from dendrograms has proved to be a highly efficient way of separating out families of faults in trials involving artificial data and natural data which has been artificially mixed.

Finally, our procedure is automatic in the sense that additional information derived from field observations regarding the approximate orientation of one or more of palaeostress tensors is not a pre-requisite for the application of the technique. This does not mean that we do not value such information. On the contrary, such information will play a vital part in future testing of the described method.

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REFERENCES

Angelier, J. 1979. Determination of the mean principal directions of stresses for a given fault population. *Tectonophysics* **56**, T17–T26.

Angelier, J. 1980. Inversion of field data in fault tectonics to obtain the regional stress III: a new rapid direct inversion by analytical means. *Geophys. J.* **103**, 363–376.

Angelier, J. 1994. Fault slip analysis and palaeostress construction. In: *Continental Deformation* (edited by Hancock, P. L.). Pergamon Press, 53–100.

Angelier, J. & Manoussis, S. 1980. Classification automatique et distinction des phases superposées en tectonique de faille. *C.r. Acad. Sci., Paris* **290**, 651–654.

Armijo, R., Carey, E. & Cisternas, A. 1982. The inverse problem in microtectonics and the separation of tectonic phases. *Tectonophysics* **82**, 145–160.

Arthaud, F. 1969. Méthode de détermination graphique des directions de raccourcissement, d'allongement et intermédiaire d'une population de failles. *Bull. Soc. géol. Fr.* **11**, 729–732.

Bott, M. H. P. 1959. The mechanics of oblique slip faulting. *Geol. Mag.* **96**, 109–117.

Carey, E. & Brunier, B. 1974. Analyse théorique et numérique d'un modélé mecanique élémentaire appliqué à l'étude d'une population de failles. *C. r. Acad. Sci., Paris* **279**, 891–894.

Etchecopar, A., Vasseur, G. & Daignieres, M. 1981. An inverse problem in microtectonics for the determination of stress tensors from fault striation analysis. *J. Struct. Geol.* **3**, 51–65.

Everitt, B. 1980. *Cluster Analysis*, Heinemann, 2nd Edn, Oxford.

Fry, N. 1992. Stress ratio determinations from striated faults: a spherical plot for cases of near-vertical principle stress. *J. Struct. Geol.* **10**, 1121–1131.

Galindo-Zaldivar, J. & Gonzalez-Lodeiro, F. 1988. Faulting phase differentiation by means of computer search on a grid pattern. *Annales Tectonicae* **2**, 90–97.

Gephart, J. W. 1990. Stress and the direction of slip on fault planes. *Tectonics* **9**, 845–858.

Gephart, J. W. & Forsyth, D. W. 1984. An improved method for determining the regional stress tensor using earthquake focal mechanism data: an application to the San Fernando earthquake sequence. *J. Geophys. Res.* **B89**, 9305–9320.

Hardcastle, K. C. 1989. Possible paleostress tensor configurations derived from fault-slip data in Eastern Vermont and Western New Hampshire. *Tectonics* **8**, 265–284.

Hardcastle, K. C. & Hills, L. S. 1991. BRUTE3 and SELECT: Quickbasic 4 programs for determination of stress tensor configurations and separation of heterogeneous populations of fault-slip data. *Computers & Geosciences* **17**, 23–43.

Huang, Q. 1988. Computer-based method to separate heterogeneous sets of fault-slip data into sub-sets. *J. Struct. Geol.* **10**, 297–299.

Jaeger, J. C. 1962. *Elasticity, Fracture and Flow*. Methuen, London.

Kleinspehn, K., Pershing, J. & Teyssier, C. 1989. Palaeostress stratigraphy: a new technique for analysing tectonic control on sedimentary-basin subsidence. *Geology* **17**, 253–257.

Lisle, R. J. 1988. Romsa: a Basic program for paleostress analysis using fault-striation data. *Computers & Geosciences* **14**, 255–259.

Michael, A. J. 1984. Determination of stress from slip data: faults and folds. *J. Geophys. Res.* **89**, 11517–11526.

Simón Gómez, J.-L. 1986. Analysis of the gradual change in stress

regime (example from the eastern Iberian Chain, Spain). *Tectonophysics* **124**, 37–53.
Will, T. M. & Powell, R. 1991. A robust approach to the calculation of paleostress fields from fault plane data. *J. Struct. Geol.* **13**, 813–821.
Wojtal, S. & Pershing, J. 1991. Paleostresses associated with faults of large offset. *J. Struct. Geol.* **13**, 49–62.

APPENDIX

Inverse stress tensor

For any given stress tensor (σ_1, σ_2, σ_3, ϕ) we define the inverse tensor as one which has co-axial orientation but with σ_1 and σ_3 interchanged and a stress difference ratio, ϕ' equalling $1 - \phi$.

Let l, m, n be the direction cosines of the plane's normal **N** referred to axes chosen to coincide with σ_1, σ_2, σ_3 axes resp, so that the components of the stress vector **S** acting on the fault plane are $\sigma_1 l$, $\sigma_2 m$, $\sigma_3 n$. Let the **O** vector be the normal of the plane which contains this stress vector X and the fault plane's normal **N**; it is equivalent to Arthaud's (1969) 'pole to the *m* plane'. The **O** vector will therefore have direction cosines proportional to

$(\sigma_3 - \sigma_2)mn$, $(\sigma_1 - \sigma_3)ln$, $(\sigma_2 - \sigma_1)lm$ (A1, see Jaeger 1962, p. 18)

Simplifying the direction ratios by setting $\sigma_3 = 0$, σ_1, $= 1$ and introducing ϕ [$= (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$] gives

$$-\phi\ mn,\ ln,\ (\phi - 1)lm \qquad \text{(A2)}$$

The **O** vector lies in the fault plane at right angles to the direction of resolved shear stress in the fault plane. The **O**′ vector for the inverse stress tensor is given by interchanging σ_1 and σ_3 and changing the stress ratio to $(1 - \phi)$

$$\phi\ mn,\ -ln,\ -(\phi - 1)lm \qquad \text{(A3)}$$

Hence for any given fault plane the **O** and **O**′ vectors for the stress tensor and its inverse, respectively, are parallel. The shear stress directions will therefore be parallel though sign convention indicates that their senses are reversed.